## Precalc Warm Up #8-5

Find all zeros, not just the real ones

1. $f(x) = x - 5$ (this first degree equation has 1 zero)

2. $f(x) = x^2 - 6x + 9$

$(x-3)^2$

$(x-3)(x-3)$

(this 2nd degree equation, counting multiplicity has 2 zeros)

3. $f(x) = x^3 + 4x$

$x(x^2+4)$

(this 3rd degree equation has 3 zeros)

$x^2 = -4$ $x = \pm 2i$

4. $f(x) = x^4 - 1$

(this 4th degree equation has 4 zeros)

$(x^2-1)(x^2+1) = (x+1)(x-1)(x^2+1)$ $x^2 = -1$ $x = \pm i$

An nth degree polynomial has at most n REAL zeros, but it has precisely n zeros and n linear factors.

October 28, 2016

HW Questions:

2. Express each of the following powers of $i$ as $i$, $-i$, 1, or $-1$.

(a) $i^{40}$

(b) $i^{25}$ $= \cancel{(i^4)^6} \cdot i^1 = i$

(c) $i^{50}$

(d) $i^{67}$ $= \cancel{(i^4)^{16}} \cdot i^3 = -i$

$\boxed{\begin{aligned} i &= i \\ i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1 \end{aligned}}$

find a and b

5. $\boxed{(a-1)} + \boxed{(b+3)}i = \boxed{5} + 8i$

$a \quad + \quad b \quad i \qquad a+bi$

$a-1=5$

$(b+3)i = 8i$

$b+3=8$

In Exercises 7–18, write the complex number in $\boxed{\text{standard}}$ form and find its $\boxed{\text{complex conjugate.}}$

$a+bi$

9. $2-\sqrt{-27}$

$2-\sqrt{-1}\sqrt{9}\sqrt{3}$

$2-\sqrt{i^2}\cdot 3\cdot\sqrt{3}$

$2-3i\sqrt{3} \leftarrow$ standard form

$2+3i\sqrt{3} \leftarrow$ complex conjugate

13. $-6i + i^2$

$-1-6i$ st. form

$-1+6i$ conjugate

**21.** $(8 - i) - (4 - i)$

**23.** $(-2 + \sqrt{-8}) + (5 - \sqrt{-50})$

**27.** $\sqrt{-6} \cdot \sqrt{-2}$

**31.** $(1 + i)(3 - 2i)$

**35.** $6i(5 - 2i)$

**39.** $(\sqrt{14} + \sqrt{10}\, i)(\sqrt{14} - \sqrt{10}\, i)$

sum and difference pattern

so difference of sq.

$(\sqrt{14})^2 - (\sqrt{10}\, i)^2$

$14 - 10i^2$

$14 + 10 = \boxed{24}$

**45.** $\dfrac{2 + i}{2 - i}$

multiply by the complex conjugate of the denominator.

$\dfrac{2+i}{2-i} \cdot \dfrac{2+i}{2+i}$

$\dfrac{4+2i+2i+i^2}{(2)^2-(i)^2}$

**53.** $\dfrac{(21 - 7i)(4 + 3i)}{2 - 5i}$

$= \dfrac{84+63i-28i-21i^2}{2-5i}$

$-21i^2$
$-21(-1)$
$+21$

$= \dfrac{105 + \cancel{91i}\ 35i}{2-5i} \cdot \dfrac{2+5i}{2+5i}$

$= \dfrac{210+525i+70i+165i^2}{(2)^2-(5i)^2}$ $= -165$

$= \dfrac{35+595i}{4-25i^2}$ $= \cdot 25$

$= \dfrac{35+595i}{29}$

In Exercises 57–64, use the quadratic formula to solve the quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**59.** $4x^2 + 16x + 17 = 0$

$$x = \frac{-16 \pm \sqrt{256 - 4(4)(17)}}{8}$$

$$x = \frac{-16 \pm \sqrt{-16}}{8}$$

$$x = -\frac{16}{8} \pm \frac{4i}{8}$$

$$x = -2 \pm \frac{1}{2}i$$

**63.** $16t^2 - 4t + 3 = 0$

follow the same process as

:)

**65.** Prove that the sum of a complex number and its conjugate is a real number.

**67.** Prove that the product of a complex number and its conjugate is a real number.

Write the polynomial as a product of linear factors, and list all of its zeros

$$f(x) = x^5 + x^4 + 8x^3 + 4x^2 - 128x - 192$$

$$\frac{\pm 1 \pm 2 \pm 3 \dots}{\pm 1}$$

| | | | | | | |
|---|---|---|---|---|---|---|
| $-2$ | 1 | 1 | 8 | 4 | $-128$ | $-192$ |
| | ↓ | $-2$ | 2 | $-20$ | 32 | 192 |
| 3 | 1 | $-1$ | 10 | $-16$ | $-96$ | 0 |
| | ↓ | 3 | 6 | 48 | 96 | |
| | 1 | 2 | 16 | 32 | 0 | |

$(x+2)(x-3)$

$$(x^3+2x^2)+(16x+32)$$

$$x^2(x+2)+16(x+2)$$

$$(x+2)(x-3)(x+2)(x^2+16)$$

$x^2+16=0$

$x^2=-16$

$\sqrt{x^2}=\sqrt{-16}$

$x=\pm 4i$

$$(x+2)^2(x-3)(x+4i)(x-4i)$$

$(x - \text{root})$

Notice that the 2 complex zeros were conjugates.

Write a 3rd degree polynomial going through (-2, 52) with zeros -4 and 1+2i. (put answer in standard form)

$(x+4)(x-(1+2i))(x-(1-2i))$

$(x+4)((x-1)-2i)((x-1)+2i)$

$(x-1)^2-(2i)^2$

$x^2-2x+1-4i^2 = -4$

pt given $(-2, 52)$

$(x+4)(x^2-2x+5)$

$x^3-2x^2+5x+4x^2-8x+20$

$y=a(x^3+2x^2-3x+20)$

What does the graph look like? Describe end behavior.

$52=a(\cancel{-8}+\cancel{8}+6+20)$

$52=26a$

$a=2$

$y=2x^3+4x^2-6x+20$

We typically factor until it is irreducible over the rationals

$(x^2+9)(x^2-3)$

Ex: factor $x^4 + 6x^2 - 27$ as the product of factors that are irreducible over the rationals.

$x^2 = -9$

$x = \pm 3i$

$(x^2+9)(x^2-3)$

Now factor it as the product of linear and quadratic factors that are irreducible over the reals

$x^2 = 3$

$x = \pm\sqrt{3}$

(x-root)

$(x^2+9)(x-\sqrt{3})(x+\sqrt{3})$

Factor it $\boxed{\text{completely}}$

$(x-3i)(x+3i)(x-\sqrt{3})(x+\sqrt{3})$

List all of the zeros. How many are there?

4

$x = 3i, -3i, -\sqrt{3}, \sqrt{3}$

Factor completely, given that one factor is $x^2 - 6$

$$\begin{array}{r} x^2-2x+3 \\ x^2-6\,\overline{\big)\,x^4-2x^3-3x^2+12x-18} \\ +(-x^4 \quad +6x^2) \\ \hline -2x^3+3x^2+12x \\ +(+2x^3 \quad -12x) \\ \hline 3x^2-18 \\ 3x^2-18 \\ \hline 0 \end{array}$$

$x^2-6=0$
$x^2=6$
$x=\pm\sqrt{6}$

$(x^2-6)(x^2-2x+3)$

take to linear factors

$(x-\sqrt{6})(x+\sqrt{6})(x-(1+i\sqrt{2}))(x-(1-i\sqrt{2}))$

$$x=\frac{2\pm\sqrt{4-4(1)(3)}}{2(1)}=\frac{2\pm\sqrt{-8}}{2}=\frac{2\pm 2i\sqrt{2}}{2}=1\pm i\sqrt{2}$$

$\sqrt{-1}\sqrt{4}\sqrt{2}$

$(x-\text{root})$

HW: PC book

p. 226 box, skip 35

Quiz Monday: PC 3.1 - 3.5